Common Logical Fallacies in Math

Using critical thinking to evaluate reasoning in math

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Objective

We will use learn about common logical fallacies in math.

We will use critical thinking skills to evaluate and support the reasoning process in mathematical problem solving.

Why do we care about critical thinking in the math classroom?

TED-Ed: A simple tool to improve critical thinking

Record the 5 tips in your notes.

Pick the one that stands out as the most interesting to you.

20 Word Gist:
Summarize why this tip is interesting, useful, confusing to you on your sticky note.

Critical Thinking Tips

Which tip did we find to be most interesting, useful, etc. overall?

1. Formulate your question
2. Gather your information
3. Apply the information
4. Consider the implications
5. Explore other points of view

Critical thinking can help us break down our thinking process into steps so that we don't jump to conclusions.

Vocabulary

Logic

(noun) the study of the formal principles of reasoning.

Evidence

something that provides proof (for a claim).

Claim

an assertion open to challenge.

Reasoning

(noun/verb) the use of logic to understand or judge something (evidence for a claim).

Logic (noun) is a set of rules which guide reasoning.

Reason (verb) is the act of using logic to understand or make a judgement.

We use claims, evidence and reasoning (CERs) to make sense of math problems

Claim $2 \times 1 = 2$

Evidence Identity Property of Multiplictation (when a number, n, is multipliedby one, the result is the number itself).

Reasoning Because of the Identity Property of Multiplication, when we multiply the number two by the number one, the result will be the number two.

Can you come up with an example?

Mathematical Properties

Answers to math problems are usually claims.

Showing your work and using mathematical properties (commutative, distributive, associative, identity) provides evidence to support your claim.

Reasoning links your claims to your evidence. We often don't provide explicit reasoning in math. **This is where critical thinking is important.**

Types of Reasoning

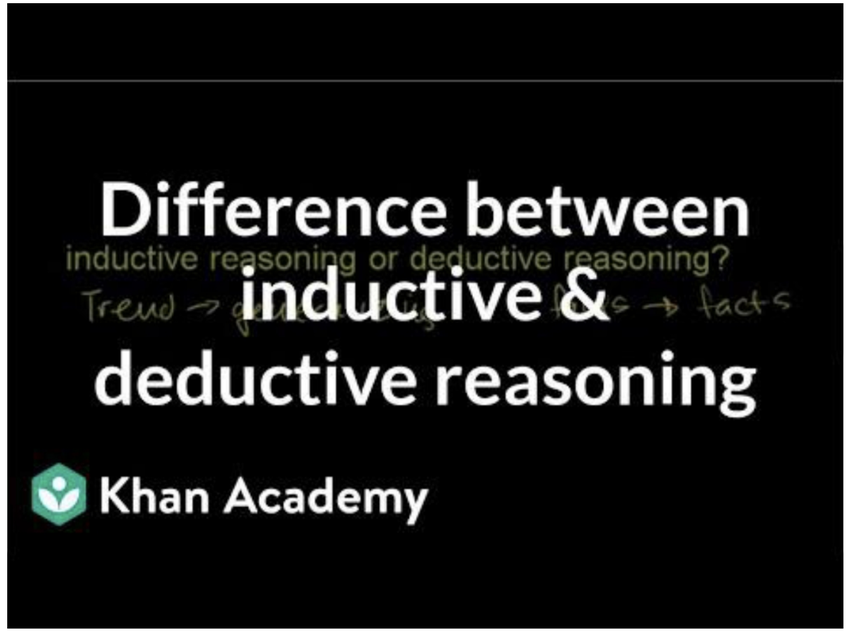
Inductive

Going from specific to general conclusions.

Deductive

Going from general to specific conclusions.

Comparing inductive and deductive reasoning



Record your own example of each type of reasoning in your notes.

Inductive Reasoning Example

Observation Dogs A & B have fleas (specific).

Pattern All observed dogs have fleas (specific).

Theory All dogs have fleas (general).

What are the weaknesses in this claim?

Using Critical Thinking to Evaluate Inductive Reasoning

If we jump from observing dogs A & B having fleas to the theory that ALL dogs have fleas, we may be experiencing the logical fallacy of **confirmation bias:**

believing or accepting only information that confirm your existing beliefs

Prevention strategies:

1. To prevent confirmation bias, we can collect additional data to test our premises.

2. We should also presume that we are wrong until we have collected enough data to provide strong support for our claim.

Preventing Confirmation Bias

Strategies

1. To prevent confirmation bias, we can collect additional data to test our premises.

2. We should also presume that we are wrong until we have collected enough data to provide strong support for our claim.

In math

1. Check your work multiple times to ensure it is error free.

2. Eliminate distractor information from word problems.

3. Compare multiple choice items carefully.

Deductive Reasoning Example

Theory	All dogs have fleas (general).
Hypothesis	All pet dogs in my neighborhood have fleas (specific).
Data	Test neighborhood dogs for fleas (specific).
Conclusion	Half didn't have fleas > reject hypothesis (specific).

What are the weaknesses in this claim?

Using Critical Thinking to Evaluate Deductive Reasoning

If we claim that all dogs have fleas based on the assumption that all dogs in the neighborhood have fleas, we may be experiencing the logical fallacy of **overgeneralization or hasty generalization bias:**

making a claim based on too little evidence

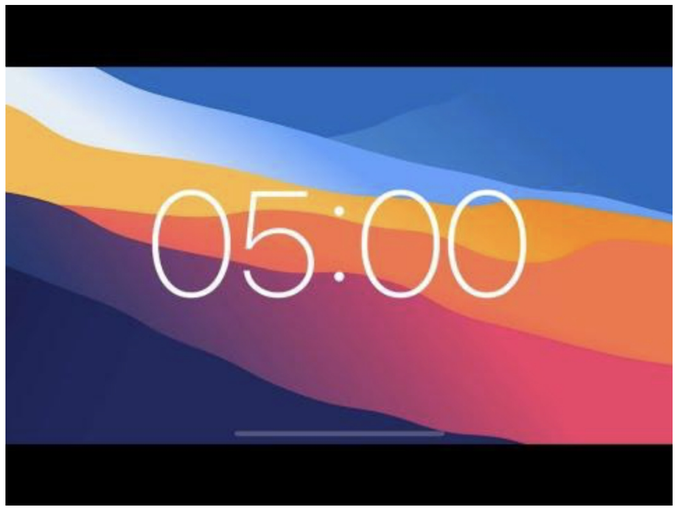
Prevention strategies:

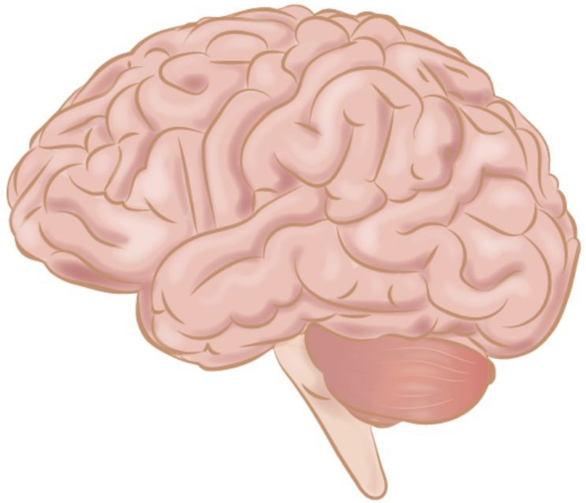
1. To prevent overgeneralization/hasty generalization bias, we can collect test our claim by collecting additional data.

2. Clearly defining our terms is also helpful to prevent this logical fallacy.

3. We should also presume that we are wrong until we have collected enough data to provide strong support for our claim.

Brain Break



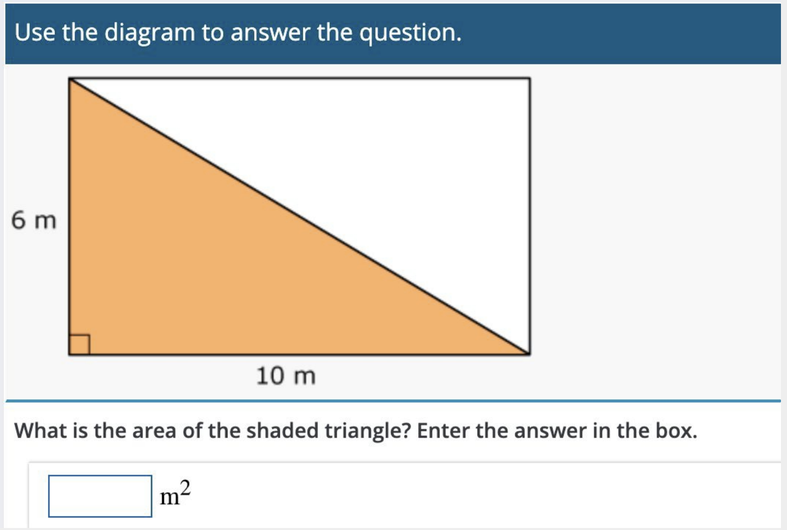


Appeal to Authority

If an authority thinks something, it must therefore be true.

Example

A student who is known to frequently score high on tests confidently states the the area is 60 square meters. Most of the class agrees that this is correct. What is the problem?



Area of a triangle is ½ bh. The student saw the rectangle and did the calculation for the area of a rectangle which is twice that of a triangle. The answer should be 30 square meters.

Problem taken from NWEA MAP Test items (2022)

Prevention

Remember that there is no "smartest person in the room." Even teachers make mistakes sometimes! Questioning authority can be a good thing.

Peer Pressure and Fallacies

Appeal to Authority is one of several fallacies driven by peer pressure. Here are two more that you might experience at school, especially during class discussion or collaborative work time:

Bandwagon bias

We adopt certain behaviors or beliefs because many other people do the same.

Groupthink bias

We let social dynamics of a group situation override the best outcome.

Prevention strategies:

1. Slow down the decision making process. Ask for additional time to think or discuss.

2. Try to make decisions in an environment where you don't feel pressured by other people.

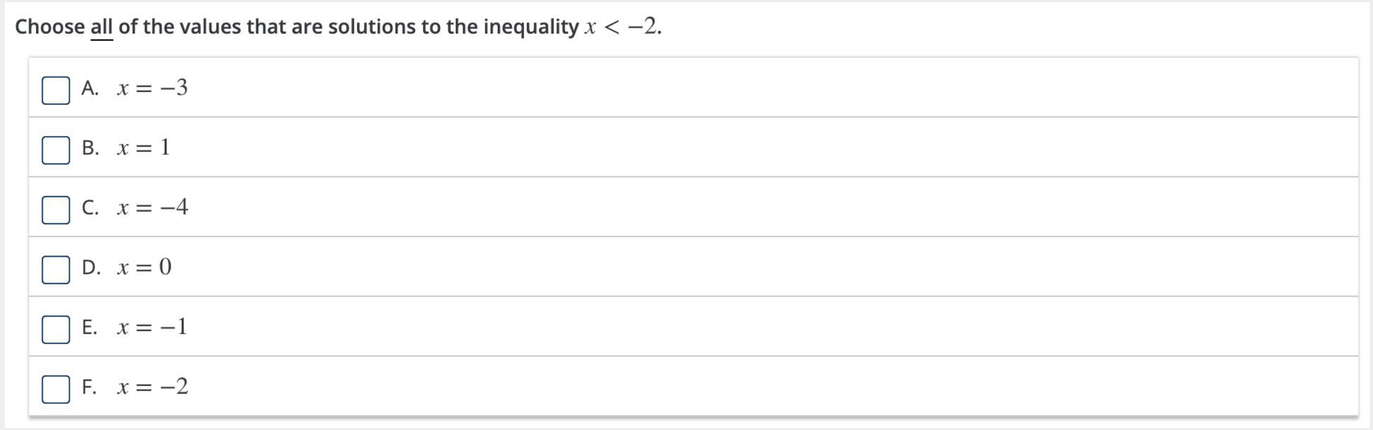
3. Consider alternative options that go against the majority view.

Availability Bias

We tend to use information that comes to mind quickly and easily when making decisions.

Example

You quickly choose A then move to the next question because you didn't notice that it said to choose ALL possible solutions. What else should you have chosen?



Problem taken from NWEA MAP Test items (2022)

Prevention

Use lists and brainstorming to make sure that all possibilities are considered. If possible, ask for outside perspectives. Read, write and draw possible solutions.

False Dilemma

Only two choices are presented, but more exist. In the prior example, the student created their own false dilemma by assuming that there was only one correct answer.

Prevention

1. Be sure to read the instructions slowly and carefully.

2. After choosing an answer, re-read the instructions to be sure that you are fully responding to the prompt.

3. Ask yourself, "is it possible that there could be more than one correct choice?"

Sunk Cost Fallacy

We tend to follow through on something when we are invested in it, whether or not the costs outweigh the benefits.

Prevention

1. Take a break! If you have reached your frustration point or are struggling to think clearly, step away from the problem for a bit and come back to it later.

2. Check your work using another strategy.

3. Get feedback from an objective source.

Class Discussion

What are some examples of these logical fallacies that could come up with this item?

Applying Critical Thinking to Math Problems

Use the information to answer the question.

There are 21 students in a class. The median age of the students is 23. The oldest student is 29.

Based on this information, what is the truthfulness of each statement in the table? Select the phrase that best describes each statement.

Statement	Truthfulness		
At least one student is 23.	Must be true	Could be true	Cannot be true
Some students are younger than 15.	Must be true	Could be true	Cannot be true

Problem taken from NWEA MAP Test items (2022)

Applying Critical Thinking to Math Problems

Step 1: Create your math plan.

The question asks for the phrase that BEST describes each statement.

Step 2: Set up your math strategy.

We aren't solving a conventional problem, so we can create a # line or draw a picture to represent the information.

Step 3: Use math strategy to determine answer.

We know there are 21 total students, meaning that the median is 23 because there is a true middle point (not an average of two middle points). We know there is at least one student who is 29. We cannot be sure if there are students under

Step 4: Ask if the answer is reasonable and double check.

- At least one student is 23: must be true because it is a true middle point due to the odd number of data.
- Some students are younger than 15: could be true because we don't have solid evidence supporting or disproving the claim.

How can we use these strategies in the future…inside and outside of the classroom?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